SIDDHARTH GROUP OF INSTITUTIONS :: PUTTUR
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QUESTION BANK (DESCRIPTIVE)

Subject with Code : Probability & Statistics (18HS0835) **Course & Branch**: MCA

Year & Sem: I-MCA & I-Sem **Regulation:** R18

Unit 1 (Probability & Random Variables)

1. (a) Define (i) Axioms of Probability (ii) Event (iii) Independent events 6 M
   (b) If $P(A)=1/2, P(B)=1/3, P(A\cap B)=1/5$ then find
   (i) $P(A\cup B)$ (ii) $P(A^c\cap B)$ (iii) $P(A\cap B^c)$ (iv) $P(A^c\cap B^c)$ 6 M
2. (a) State and Prove Addition theorem on Probability 6 M
   (b) State and Prove Multiplication theorem of Probability. 6 M
3. (a) The probabilities that students A, B, C, D solve a problem are $1/3, 2/5, 1/5$ and $1/4$ respectively. If all of them try to solve the problem, what is the probability that the problem is solved. 6 M
   (b) A can hit a target 3 times in 5 shots, B hits target 2 times in 5 shots, C hits target 3 times in 4 shots. Find the probability of target being hit when all of them try. 6 M
4. State and Prove Baye's theorem 12 M
5. In a bolt factory machines M1, M2, M3 manufacture 20%, 30% and 50% of the total of their output and 6%, 3% and 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is manufactured from (i) Machine M1 (ii) Machine M2 (ii) Machine M3. 12 M
6. A business man goes to hotels X, Y, Z, 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbings. What is the probability that business man's room having faulty plumbing is assigned to hotel z. 12 M
7. (a) The probability density function of a variate X is as follows:

x	-3	-2	-1	0	1	2	3
p(x)	K	0.1	K	0.2	2K	0.4	2K

   Determine (i) K (ii) $P(X<1)$, $P(X\geq 1)$ and $P(0<X\leq 3)$ (iii) The distribution function (iv) Mean, Variance and Standard Deviation 6 M
   (b) For the following probability distribution

x	-3	-2	-1	0	1	2	3
p(x)	0.001	0.01	0.1	0.2	2K	0.4	2K

   Then find (i) K (ii) Mean and Variance (iii) E (X^2+2X+3) 6 M

8. (a) A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. Find the expected number of defective items and variance of defective items. 6 M

(b) Let X and Y be the independent variables with E(X) = 3, E(Y) = 10, $E(X^2) = 25$, $E(Y^2) = 164$ then find (i) E (3X+Y-8) (ii) E(2X-3Y+7) (iii) V(X) and V(Y) (iv) V(3X+Y-8) and V(2X-3Y+7) 6 M

9. (a) The probability density $f(x)$ of a continuous random variable is given by $f(x) = \begin{cases} k(1-x^2), \text{ for } 0 < x < 1 \\ 0, \text{ elsewhere} \end{cases}$ (i) Find k (ii) Find the probability between 0.1 and 0.2 (iii) greater than 0.5 6 M

(b) For the continuous probability function $f(x) = k\,x^2 e^{-x}$ when $x \geq 0$, find (i) k (ii) Mean (iii) Variance 6 M

10. A continuous random variable has the probability density function $f(x) = k\,e^{-|x|}, -\infty < x < \infty$ (i) Find k (ii) Mean and Variance (iii) Find the probability that the variate lies between 0 and 4 (iii) greater than 0.5 12 M

11. The probability density $f(x)$ of a continuous random variable is given by

$f(x) = \begin{cases} 2e^{-2x}, \text{ for } x > 0 \\ 0, \text{ for } x \leq 0 \end{cases}$, find the probabilities that it will take on a value

(i) between 1 and 3 (ii) greater than 0.5 12 M

Unit-2 (Distributions)

1 Find the Mean and Variance of the Binomial distribution 12 M

2 (a)Ten coins are thrown simultaneously. Find the probability of getting at least seven heads6 M

(b) The mean and variance of a binomial variable X with parameters n and p are 16 and 8. Find $P(X \geq 1)$ and $P(X > 2)$ 6 M

3 (a) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 success are 0.4096 and 0.2048. Find the parameter 'p' of the distribution. 6 M

(b) Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys? Assume equal probabilities for boys and girls. 6 M

4 Find the Mean and Variance of the Poisson Distribution 12 M

5 (a) 2 % of the items of a factory are defective. The items are packed in boxes. What is the probability that there will be (i) 2 defective items (ii) at least three defective items in a box of 100 items. 6 M

(b) A hospital switch board receives an average of 4 emergency calls in a 10 minute interval. What is the probability that (i) there are at most 2 emergency calls in a 10 minute interval (ii) there are exactly 3 emergency calls in a 10 minute interval. 6 M

6 (a) If a Poisson distribution is such that P(X=1) $\frac{3}{2}$ = P(X=3) then find (i) $P(X \geq 1)$ (ii) $P(X \leq 3)$ (iii) $P(2 \leq X \leq 5)$ 6 M

(b) If x is a Poisson variate such that $3p(x=4) = \frac{1}{2}p(x=2) + p(x=0)$ find (i) the mean of x (ii) $P(X \leq 2)$ 6 M

7 Find the Mean and Variance of a Normal Distribution 12 M

8 (a) What are the main Characterstic equations of the Normal Distribution 6 M

(b) For a normally distributed variate with mean 1 and standard deviation 3, find the probabilities that (i) $3.43 \leq x \leq 6.19$ (ii) $-1.43 \leq x \leq 6.19$ 6 M

9 (a) In a Normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and standard deviation of the distribution. 6 M

(b) If X is a normal variate, find the area A (i) to the left of z = -1.78 (ii) to the right of z = -1.45 (iii) corresponding to $-0.8 \leq z \leq 1.53$ 6 M

10 The marks obtained in Mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%.Determine

(i) How many students got marks above 90%

(ii) What was the highest mark obtained by the lowest 10% of the students

(iii) Within what limits did the middle of 90% of the students lie. 12 M

11 Given that the mean heights of the students in a class is 158 cms with standard deviation of 20 cms. Find how many students heights lie between 150 cms and 170 cms, if there are 100 students in the class 12 M

Unit-3 (Basic Statistics)

1. (a) Define Moments, Skewness and Kurtosis 6 M

   (b) Find Pearson's coefficient of skewness for the following data 12 M

Class	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
Frequency	5	9	14	20	25	15	8	4

2. Calculate the median, quartiles and the quartile coefficient of skewness from the following data 12 M

Weights(lbs)	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of persons	12	18	35	42	50	45	20	8

3. Compute the quartile coefficient of skewness from the following distribution 12 M

x	3-7	8-12	13-17	18-22	23-27	28-32	33-37	38-42
f	2	108	580	175	80	32	18	5

4. Psychological tests of intelligence and of engineering ability were applied to 10 students. Here is a record of ungrouped data showing intelligence ratio (I. R) and engineering ratio (E.R). Calculate the co-efficient of correlation. 12 M

Students	A	B	C	D	E	F	G	H	I	J
I.R	105	104	102	101	100	99	98	96	93	92
E.R	101	103	100	98	95	96	104	92	97	94

5. Find the rank correlation for the following data 12 M

x	56	42	72	36	63	47	55	49	38
y	147	125	160	118	149	128	150	145	115

6. Find the Correlation coefficient between x and y from the given data 12 M

x	78	89	97	69	59	79	68	57
y	125	137	156	112	107	138	123	108

7. (a) If θ is the angle between the two regression lines then show that

   $Tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$, Explain the significance when $r = 0$ and $r = \pm 1$ 6 M

   (b) For two random variables x and y with the same mean, the two regression lines are

   $y = ax + b$ and $x = cy + d$. Show that $\frac{b}{d} = \frac{1-a}{1-c}$ 6 M

8. For the following data determine (i)least squares regression line of y on x (ii) y(3) (iii) least squares regression line of x on y (iv) x(4) 12 M

x	6	5	8	8	7	6	10	4	9	7
y	8	7	7	10	5	8	10	6	8	6

9. Establish the formula $r = \frac{\sigma_x^{\,2} + \sigma_y^{\,2} - \sigma_{x-y}^{\,2}}{2\sigma_x\sigma_y}$, Hence calculate r from the following data 12 M

x	21	23	30	54	57	58	72	78	87	90
y	60	71	72	83	110	84	100	92	113	135

10. (a) Define lines of regression and regression coefficients (i) x on y (ii) y on x 6 M

(b) The two regressions of the variables x and y are x = 19.13 – 0.87 y and y = 11.64 – 0.50 x.

Find (i) mean of x's (ii) mean of y's (iii) the correlation coefficient between x and y . 6 M

Unit-4 (Curve Fitting &Test of Hypothesis)

1. (a) Find the normal equations for a straight line by using the method of least squares 6 M
   (b) Find the normal equations for a second degree parabola by using the method of least squares 6 M

2. By the method of least squares find the straight line that best fits from the following data

x	0	5	10	15	20	25
y	12	15	17	22	24	30

   And also find y(13) 12 M

3. Fit a second degree polynomial (or) parabola from the following data by using the method of least squares

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

   And also find y(3.5) 12 M

4. Use the method of least squares find the exponential curve $y = ae^{bx}$ from the following data

x	1	5	7	9	12
y	10	15	12	15	21

   Also find y(11) 12 M

5. Fit the curve $y = ab^x$ from the following data by using the method of least squares

x	2	3	4	5	62
y	8.3	15.4	33.1	65.2	127.4

6. (a) Define the null hypothesis, alternate hypothesis 6 M
   (b) In a sample of 1000 people in Karnataka 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance? 6 M

7. Experience had shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level and find confidence limits. 12 M

8. (a) the mean and standard deviation of a population are 11795 and 14054 respectively. If n=50 find 95% confidence limits for mean 6 M
   (b) In two large populations, there are 30% and 225% respectively of fair haired people. In this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations. 6 M

9. It is claimed that a random sample of 49 tyres has a mean life of 15200 km. This sample was drawn from a population whose mean is 15150 kms and a standard deviation of 1200 km. Test the significance at 0.05 level and also find 95% confidence limits. 12 M

10. (a) The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same Population of S.D 2.5 inches 6 M

    (b) Samples of students were drawn from two universities and from their weights in kilograms, mean and standard deviations are calculated and shown below. Make a large sample test to test the significance of the difference between the means 6 M

	Mean	S.D	Size of the sample
University A	55	10	400
University B	57	15	100

UNIT 5 (Test of Significance)

1. (a) write about working rule for test of hypothesis for small samples 5 M

   (b) The mean life of a sample of 25 fluorescent lights bulbs produced by a company is computed to be 1570 Hours with a standard deviation of 120 Hours. The company claims that the average life of the bulbs produced by the company is 1600 Hours. Using the level of significance of 0.05, is the claim acceptable? 5 M

2. (a) A random sample of 10 boys had the following I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100, Do these data support the assumption of a population mean I.Q of 100?. Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie. 5 M

   (b) Define Critical region, Type-I and Type-II errors 5 M

3. Below are given the gain in weights (in lbs) of pigs fed on two diets A and B

Diet A	25	32	30	34	24	14	32	24	30	31	35	25	-	-
Diet B	44	34	22	10	47	31	40	32	35	18	21	35	29	22

   Test if the two diets differ significantly as regards their effect on increase in weight 10 M

4. Two independent samples of 7 items respectively had the following values.

Sample I	11	11	13	11	15	9	12	14
Sample II	9	11	10	13	9	8	10	-

   Is the difference between the means of samples significant? 10 M

5. (a) Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins, show the sample standard deviation of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal, test the hypothesis that the two variances are equal? 5 M

   (b) The nicotine in milligrams of two samples of tobacco were found to be as follows. Find the standard error and confidence limits for the difference between the means at 0.05 level. 5M

Sample A	24	27	26	23	25	-
Sample B	29	30	30	31	24	36

6. To examine the hypothesis that the husbands are more intelligent than the wives, an investigator took a sample of 10 couples and administered them a test which measures the I.Q. The results are as follows:

Husbands	117	105	97	105	123	109	86	78	103	107
Wives	106	98	87	104	116	95	90	69	108	85

   Test the hypothesis with a reasonable test at 0.05 L.O.S 10 M

7. The following figure show the distribution of digits in numbers chosen at random from a telephone directory

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	1075	933	1107	972	964	853

Test weather the digits may be taken to occur equally frequently in the directory 10 M

8. A pair of dice are thrown 360 times and the frequency of each sum is indicated below

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	8	24	35	37	44	65	51	42	26	14	14

Would you say that the dice are fair on the basis of the chi-square test at 0.05 level of significance? 10 M

9. On the basis of information given below about the treatment of 200 patients suffering from a disease, state weather the new treatment is comparatively superior to the conventional treatment. 10 M

	Favourable	Not favourable	Total
New	60	30	90
Conventional	40	70	110

10. Four methods are under development for making discs of a super conduction material. Fifty discs are made by each method and they are checked for super conductivity when cooled with liquid.

	1 st Method	2 nd Method	3 rd Method	4 th Method
Super Conductors	31	42	22	25
Failures	19	8	28	25

Test the significant difference between the proportions of super conductors at 0.05 level. 10 M

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